The Multi-attribute Elimination-By-Aspects (MEBA) Model

by

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Certificate of Authorship / Originality

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text. I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Author . . . .

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Abstract

Our research proposes a new, multi-attribute, parameterisation of Tversky's Elimination-By-Aspects (EBA) model. The EBA model conceptualises choice as a covert sequential elimination process with choice probabilities formulated over all consideration sets of the choice set. This specification attempts to capture the effect of context on choice behaviour. However, the EBA model has seen limited usage due to the large number of required parameters given the set of items under study. For a set of items T, it has $2^{|T|} - 3$ free parameters, which is infeasible for all but the simplest of contexts. To provide a practical operationalisation, we impose a set of *a priori* constraints on the parameter space. We define a generic multi-attribute structure to the set of aspects. This restricts the cardinality of the set of unknown scale values while retaining the functional (recursive) form of the model. The EBA hypothesis of a population of lexicographic decision-makers can therefore be tested in more market-realistic contexts, and inferences made over a large universal set of items described by the complete factorial. We call this model the Multi-attribute Elimination-By-Aspects (MEBA) model. The MEBA model reduces the set of unknown free parameters to a maximum of $|T| - 1$. We develop a general algebraic expression for the MEBA choice probabilities as a function of the attributes of the options in the choice set. This enables the derivation of a likelihood function, and consequently maximum likelihood estimation. We also consider the form of optimal MEBA paired comparison designs. Using Monte Carlo simulation and a discrete choice experiment with consumers, we conduct an initial empirical test of the model against the special case of the MNL model (that assumes no context effects) and find the MEBA model to be a better approximation of observed choice behaviour. This is achieved on a common set of parameters, and so it is due solely to the difference in functional form of the two models. We conclude with a discussion on future research directions, in particular the introduction of heterogeneity into the model, and the description of optimal choice experiments for larger choice set sizes.

Thesis Supervisor: Professor Jordan J Louviere

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$\varnothing$	Empty set.
2^A	Power set (set of all sets) of A.
$\binom{a}{b}$, read 'a choose b'	Number of ways of choosing b objects from a set of a distinct objects, without repetitions.
$\times$	Cartesian product.
$\otimes$	Matrix direct product.
$\mathbf{0}_{m,n}$	$m \times n$ zero matrix.
$\mathrm{diag}(x_1, \ldots, x_n)$	Matrix with ith diagonal entry x_i, off-diagonal entries 0.
$\wedge$	Logical AND.
$\neg$	Logical NOT.
$\vee$	Logical OR.
$A, B, \ldots \subseteq T$	Non-empty consideration sets of the choice set T.
$A_i, B_j, \ldots$	Variables ranging over non-empty consideration sets of T.
$\lvert A \rvert$	Cardinality (number of elements) of the set A.
$A \setminus B$	Set of items which belong to A but not to B.
A'	Set of aspects that belong to at least one option in A.
A°	Set of aspects that are common too all the options in A.
A^α	Subset of options of A that have aspect α.
$\overline{A}_i$	Set of aspects that are included in all the options in A_i, and not included in any of the options that do not belong to A_i.

$A_{(i_1,i_2,\ldots,i_m)}$	The consideration set $A = \{\boldsymbol{x}_{i_1}, \boldsymbol{x}_{i_2}, \ldots, \boldsymbol{x}_{i_m}\}$.
$a_{\boldsymbol{v}_j}$	$i_{\boldsymbol{v}_j}/N$.
$a_{\boldsymbol{d}}$	$i_{\boldsymbol{d}}/N$.
$(b_1, b_2, \ldots, b_{\lvert B_j\rvert})$	Indices for the options of the choice set that comprise the consideration set B_j.
C_M	Information matrix $\mathscr{I}(\boldsymbol{\beta})$ for the MNL main effects only model.
$c_{\boldsymbol{v}_j}$	Number of choice sets containing the treatment combination $00\ldots0$ with difference vector $\boldsymbol{v}_j$.
$\boldsymbol{d} = d_1d_2\ldots d_k$	Binary k-tuple difference vector entry.
$\boldsymbol{d}_{(i_1,i_2,\ldots,i_m)}$	Binary k-tuple difference vector entry for the choice set $\{\boldsymbol{x}_{i_1}, \boldsymbol{x}_{i_2}, \ldots, \boldsymbol{x}_{i_m}\}$.
$d_{(i_1,i_2,\ldots,i_m),r,q}$	Binary indicator for the difference vector entry $\boldsymbol{d}_{(i_1,i_2,\ldots,i_m)}$ and choice set $\{\boldsymbol{x}_{i_1}, \boldsymbol{x}_{i_2}, \ldots, \boldsymbol{x}_{i_m}\}$.
$E = \{e_1, e_2, \ldots\} = \{1, 2, \ldots\}$	Set of elimination steps.
$\boldsymbol{f}_j = (f_{j1}, f_{j2}, \ldots, f_{j\lvert\boldsymbol{f}_j\rvert})$	Vector recording the attribute indices that comprise the factorial effect $\boldsymbol{f}_j$.
$(\boldsymbol{f}_j)_{r_1,r_2,\ldots,r_{\lvert\boldsymbol{f}_j\rvert}}$	Simple effect for the treatment combination $r_1r_2\ldots r_{\lvert\boldsymbol{f}_j\rvert}$.
$H(\boldsymbol{\theta})$	Hessian matrix.
$h_{i_1,i_2,\alpha}(w_{i_1,i_2,\alpha}, \boldsymbol{\beta})$	Probability density function for subject α and choice set $(\boldsymbol{x}_{i_1}, \boldsymbol{x}_{i_2})$ in a paired comparison experiment.
iff	Abbreviation for if and only if.
$\boldsymbol{I}_n$	Identity matrix of order n.

$i_{\boldsymbol{v}_j}$	An indicator variable which takes the value one if all the choice sets with the difference vector $\boldsymbol{v}_j$ appear in the choice experiment, and zero otherwise.
$\boldsymbol{j}'_{\ell_q}$	$1 \times \ell_q$ vector of ones.
k	Number of attributes.
ℓ_q	Number of levels of the qth attribute, $(0, 1, \ldots, \ell_q - 1)$.
L	Number of items in the universal set $\mathscr{U}$.
$L(\boldsymbol{\theta})$	Likelihood function.
m	Choice set size.
N	Number of choice sets in the experiment.
n_{i_1,i_2}	Indicator variable which takes the value one when $(\boldsymbol{x}_{i_1}, \boldsymbol{x}_{i_2})$ is a choice set in the experiment, or zero otherwise.
$O_i, \quad i = 1, 2, \ldots, \lvert\mathscr{U}\rvert$	The ith item in the universal set.
$P(x, A)$	Probability of choosing item x from the consideration set A.
$P(x; y)$	Abbreviation for $P(x, \{x, y\})$.
$\boldsymbol{p}(\ell_q, x_{(i_1,i_2,\ldots,i_m)_j,q})$	Vector of orthogonal polynomial coefficients for the value $x_{(i_1,i_2,\ldots,i_m)_j,q}$ where the qth attribute has ℓ_q levels.
$\boldsymbol{p}^*(\ell_q, x_{(i_1,i_2,\ldots,i_m)_j,q})$	Normalised vector of orthogonal polynomial coefficients for an attribute with ℓ_q levels.

$Q_A(B, \boldsymbol{s}, e_j)$	Probability that decision makers described by characteristics $\boldsymbol{s}$ when starting with consideration set A will reach set B through the elimination of the options in $A \setminus B$ at step e_j. A transition probability.
$Q_A(B)$	A transition probability not conditioned on $\boldsymbol{s}$ and e_j.
$R(1, A)$	Set of rankings of T in which option 1 is ranked above all other options of $A = \{1, \ldots, \|A\|\} \subseteq T$.
r_A	Ranking of the set A.
${r_B}^A$	Ranking of B, $A \subseteq B \subseteq T$, whose restriction to A is r_A.
$\boldsymbol{s} \in \mathscr{S}$	Vector of characteristics of a decision maker.
s	Number of subjects in the experiment.
S_q	The least upper bound for the sum of the differences for a particular attribute q in a choice set containing m options.
$T \subseteq \mathscr{U}$	Choice set of items defined exogenously to the choice process.
$u(\alpha)$	The scale (merit) of the aspect α. A real-valued, non-negative function.
$u(\boldsymbol{f}_j, (i_1, i_2, \ldots, i_m), n)$	The scale value for the factorial effect $\boldsymbol{f}_j$.
$U(A)$	The unique advantage of the consideration set A. A real-valued, non-negative function.
$U((i_1, i_2, \ldots, i_m), (b_1, b_2, \ldots, b_{\|B_j\|}))$	The unique advantage of the consideration set $(b_1, b_2, \ldots, b_{\|B_j\|})$ given the choice set $\{\boldsymbol{x}_{i_1}, \boldsymbol{x}_{i_2}, \ldots, \boldsymbol{x}_{i_m}\}$.

U_{b_1}	Abbreviation for $U((i_1, i_2, \ldots, i_m), (b_1, b_2, \ldots, b_{\lvert B_j \rvert}))$ when $m = 2$.
$\boldsymbol{u}_j$	Vector recording the $m-1$ index positions of the difference vector entries in $\boldsymbol{v}$ that refer to each of the jth options.
$\boldsymbol{U} = (\boldsymbol{U}_x, \ldots, \boldsymbol{U}_z)$	Random utility vector.
$\boldsymbol{v}$	Difference vector.
$\boldsymbol{v}_{(i_1, i_2, \ldots, i_m)}$	Difference vector for the choice set $\{\boldsymbol{x}_{i_1}, \boldsymbol{x}_{i_2}, \ldots, \boldsymbol{x}_{i_m}\}$.
$w_{i_1, i_2, \alpha}$	Indicator variable which takes the value 1 when $\boldsymbol{x}_{i_1}$ is preferred to $\boldsymbol{x}_{i_2}$, $i_1 \neq i_2$, for subject α in a paired comparison experiment.
$w_{(\ell_{q_1}, \ell_{q_2}, \ldots), r}$	Normalisation constant for the rth order polynomial for the interaction of attributes with $\ell_{q_1}, \ell_{q_2}, \ldots$ levels.
$W_{(\ell_{q_1}, \ell_{q_2}, \ldots)}$	Matrix of order $(\ell_{q_1} - 1)(\ell_{q_2} - 1) \ldots$ of normalisation constants.
$x, y, z, \ldots$	Choice items
$x' = \{\alpha, \beta, \ldots\}$	Aspects of item x.
X'	The set of aspects for the universal set $\mathscr{U}$.
$\boldsymbol{x}_j = (x_{j1}, x_{j2}, \ldots, x_{jk})$	Vector of k attribute levels describing the jth item in the choice set.
$\boldsymbol{x}'$	Aspects of item $\boldsymbol{x}$.
$x_{\boldsymbol{v}_j; \boldsymbol{d}}$	The number of times the difference $\boldsymbol{d}$ appears in the difference vector $\boldsymbol{v}_j$.

$\boldsymbol{z}_{(i_1,i_2,\ldots,i_m),b_1,\boldsymbol{f}_j}$	The orthogonal polynomial coefficients for the $x_{b_1 f_{j1}} \times x_{b_1 f_{j2}} \times \ldots \times x_{b_1 f_{j\lvert \boldsymbol{f}_j \rvert}}$ effect for option b_1 in the choice set $\{\boldsymbol{x}_{i_1}, \boldsymbol{x}_{i_2}, \ldots, \boldsymbol{x}_{i_m}\}$.
$\mathscr{A}$	$\{A_k \lvert A_k \cap A \neq A, \varnothing\}$.
$\mathscr{B}$	$\{B_j \lvert B_j \cap A \neq A, \varnothing \text{ \& } B_j \cap \{x\} \neq \varnothing\}$.
$\mathscr{E}(.)$	Expectation value.
$\mathscr{F}$	Set of factorial effects of interest.
$\mathscr{I}(\boldsymbol{\theta})$	Information matrix for parameter vector $\boldsymbol{\theta}$.
$\mathscr{N}$	Fundamental matrix.
$\mathscr{P}$	Matrix of absorption probabilities.
$\mathscr{Q}$	Transition matrix.
$\mathscr{R}$	Matrix of absorbing states.
$\mathscr{S}$	Universe of vectors of measured attributes of decision-makers.
$\mathscr{T}$	Matrix of transient states.
$\mathscr{U}$	Universal set of choice items.
$(\alpha)_i, (\beta)_j, (\alpha\beta)_{ij}$	Factorial effects.
$\boldsymbol{\beta}$	Vector of coefficients of a linear function.
$\boldsymbol{\beta}_{\boldsymbol{f}_j}$	Vector of unknown coefficients associated with the jth factorial effect.
$\delta_{(b_1,b_2,\ldots,b_{\lvert B_j \rvert}),\boldsymbol{f}_j}$	$\prod_{t=1}^{m-1} \left(\lvert \kappa_{(b_1,b_2,\ldots,b_{\lvert B_j \rvert}),t} - \mu_{(b_1,b_2,\ldots,b_{\lvert B_j \rvert}),\boldsymbol{f}_j,t} \rvert \right)$.
$\delta_{\boldsymbol{f}_j}$	Abbreviation for $\delta_{(b_1,b_2,\ldots,b_{\lvert B_j \rvert}),\boldsymbol{f}_j}$ when $m = 2$.
γ_i	Systematic utility of item O_i.
$\kappa_{(b_1,b_2,\ldots,b_{\lvert B_j \rvert}),t}$	An indicator which records whether a similarity or difference is needed to create a partition of the choice set along the qth attribute which includes the consideration set $(b_1, b_2, \ldots, b_{\lvert B_j \rvert})$.

$\Lambda(\boldsymbol{\pi})$	Alternative notation for the information matrix $\mathscr{I}(\boldsymbol{\gamma})$.
λ_{i_1,i_2}	$\frac{n_{i_1,i_2}}{N}$.
$\mu_{(b_1,b_2,\ldots,b_{\lvert B_j\rvert}),\boldsymbol{f}_j,t}$	$1-\left(\prod_{p=1}^{\lvert\boldsymbol{f}_j\rvert}\left(1-d_{(i_1,i_2,\ldots,i_m),(u_{b_1,t}),f_{jp}}\right)\right)$.
MEBA(j_1,j_2,...)	A MEBA model where the factorial effects $\boldsymbol{f}_{j_1}$, $\boldsymbol{f}_{j_2}$, ... have been defined as possible sources of substitutability.
$\nu_{(b_1,b_2,\ldots,b_{\lvert B_j\rvert})}$	An indicator for whether the consideration set $(b_1,b_2,\ldots,b_{\lvert B_j\rvert})$ contains a single option and not all factorial effects have been defined as possible sources of substitutability.
$\pi(x,A)$	Permutation of 2^T.
π_i	Merit of item O_i.
ρ_j	An indicator for whether the jth factorial effect is defined as a possible source of substitutability.
$\sigma_{(i_1,i_2)}$	Variance of the MEBA choice probability $P(\boldsymbol{x}_{i_1};\boldsymbol{x}_{i_2})$.
$\boldsymbol{\theta}$	An arbitrary vector of parameters.

"The EBA functional form has considerable potential for econometric applications.... One drawback... is that the motivation for the model provides little guidance for parametric specification of the scale function. . . ."

McFadden (1981) p.226

"Application of the model to market choice would require that [the scales] be made parametric functions of the measured attributes of the alternatives."

McFadden (1980) p.S18

"The addition of such assumptions [on the structure or the relative weights of aspects] strengthens the predictions of the model and tightens its empirical interpretation."

Tversky (1972a) p.297